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WEAPONS SYSTEMS EVALUATION GROUP
ODDR&E
Washington 25, D. C.

SECOND VOLUME OF

ENCLOSURE "A"

EVALUATION OF PROGRAMMED STRATEGIC OFFENSIVE SYSTEMS 1964-1967

WSEG REPORT NO. 50

27 December 1960

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Second Volume of
Enclosure "A"
WSEG Report No. 50

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APPENDIX "B" TO ENCLOSURE "A"

MEASURE OF EFFECTIVENESS FOR "POWER LAW" TARGETS

APPENDIX "B" TO ENCLOSURE "A"

MEASURE OF EFFECTIVENESS FOR "POWER LAW" TARGETS

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MEASURE OF EFFECTIVENESS FOR "POWER LAW" TARGETS

INTRODUCTION

The purpose of this Appendix is to describe a measure of effectiveness for weapon systems employed against isolated point targets, or more generally, when employed against those categories of area targets or target complexes which obey the "power law". A target obeys the "power law" if the expected residue surviving a multiple weapon attack can be expressed as the product of the expected surviving residues from each weapon delivered singly.[1/]

This measure is an improvement over measures that assume infinite divisibility of weapons. By including the constraint that weapons may only be delivered in integral numbers, the measure is more precise, and in particular avoids the large errors which can otherwise occur in the case of kill probabilities approaching unity.

DERIVATION

The present measure is based upon a determination of the actual number of weapons (in inventory) required to produce a specified amount of damage against a specified type of target. The inverse of the number so determined is then chosen as the measure of effectiveness (MOE). The measure thus has the following direct significance -- the cost of the weapon divided by this measure of its effectiveness is simply the total cost for obtaining the desired effect on a single target of specified type.

1/ Many area targets (such as urban floor space, blast casualties, ect.) for typical cities appear to follow the power law reasonably well, provided an optimal pattern of desired ground zeros is chosen.

To begin the derivation, consider the case of M missiles attacking N identical targets. Let S be the probability that a target survives any single attacking missile (the single-shot survival probability). Let $K = [M/N]$ represent the "basic coverage," where the bracket symbol is such that $[x]$ denotes the largest integer contained in x. K, therefore, represents the largest number of weapons which can be applied to every target without exceeding the total available number of weapons M. Let $L = M - NK$ denote the "excess," the number of weapons left over after applying the basic coverage to each target. These excess missiles can then be applied to L of the targets, which then have a total $K + 1$ weapons, while the remaining $N - L$ targets have had just K weapons applied to them. The expected fraction of the target system surviving is, therefore,

$$E = LS^{K+1} + (N - L)S^K = \bar{S}N \tag{1}$$

where E is the expected number of targets in the target system surviving the attack and $\bar{S}$ is the expected fraction of the total target system surviving the attack. Equation (1) can be rewritten in the form

$$\bar{S} = S^K (1 - (L/N)(1 - S)) \tag{2}$$

If we let $\beta = L/N (0 \leq \beta < 1)$ represent the fraction of the target system to which $K + 1$ weapons are applied, and let $\alpha = K + \beta$ then $\alpha = M/N$ represents the average number of weapons which must be delivered against each target in the target system to achieve the desired overall effect.

Its reciprocal is the desired measure of effectiveness we seek:

$$MOE = 1/\hat{a}. \tag{3}$$

In order to determine α Equation (2) may be rewritten in the form:

$$\overline{S} = S^K(1 - \beta(1 - S)) \tag{4}$$

which must now be solved for K and β.

Consider for the moment the new equation

$$\overline{S} = S^{K'} \tag{5}$$

where the parameter K' is allowed to take on any real number. If we identify K' with the quantity $K + \beta$ it will be observed that Equations (4) and (5) coincide for all integral values of K'. Now Equation (5) has the solution

$$K' = \ell n\, \overline{S}/\ell n\, S. \tag{6}$$

We can now determine the parameter K in Equation (4). Because Equations (4) and (5) agree for integral values of K' it follows that K must be the largest integer contained in K', so that

$$K = [\ell n\, \overline{S}/\ell n\, S]. \tag{7}$$

It remains now only to determine the value of β. Rearrangement of Equation (4) leads to the solution for β:

$$\beta = (1 - \overline{S}/S^K)/(1 - S). \tag{8}$$

The formulas may be simplified by introducing the following new notation:

$$\mu = \ell n\, \overline{S}/\ell n\, S$$
$$\delta = \mu - [\mu]. \tag{9}$$

μ represents the theoretical number of weapons which would be required if the weapons were infinitely divisible. It is

identical to the earlier measure used occasionally by WSEG. δ represents the departure of this theoretical number from integrality. From Equation (7) we have that $K = [\mu] = \mu - \delta$, so that

$$\alpha = K + \beta = \mu - \delta + (1 - \bar{S}/S^{\mu-\delta})/(1 - S) \tag{10}$$

which is simply reduced to the form

$$\alpha = \mu - \delta + (1 - S^{\delta})/(1 - S) . \tag{11}$$

The term $(1 - S^{\delta})/(1 - S) - \delta$ in Equation (11) can be regarded as the term which corrects the measure of effectiveness for the effects of the constraint to integral numbers of weapons.

Summarizing the preceding, the following set of equations is to be used to determine the measure of effectiveness:

$$\mu = \ln \bar{S}/\ln S \tag{12a}$$

$$\delta = \mu - [\mu] \tag{12b}$$

$$\alpha = \mu - \delta + (1 - S^{\delta})/(1 - S) \tag{12c}$$

INCLUSION OF OPERATIONAL FACTORS

The inclusion of the operational factors of reliability, penetrability, survivability, and reprogrammability, will now be developed. We shall recognize three subdivisions of the operational reliability of a weapon system, the on-launcher reliability denoted by RL, The inflight reliability denoted by RF, and the terminal reliability denoted by RT. The on-launcher reliability, RL, denotes the probability that a missile on-launcher will be successfully counted down to the point of lift-off. The inflight reliability, RF, denotes the probability that a missile will perform without malfunction through the burning phase, and terminal reliability, RT, denotes the probability that the missile will successfully re-enter and detonate. In addition, we shall include the effect of the penetrability, P, the probability that the delivery vehicle

will successfully penetrate any active defenses, and the survivability, Q, the probability that the delivery vehicle survives an enemy attack prior to its launch. Finally, we shall treat three degrees of reprogrammability for the weapon systems, "full" reprogrammability -- the case in which all failures prior to burn-out are detectable and correctable by the retargeting of subsequent missiles, partial reprogramming -- the case when only those failures which occur prior to lift-off are detectable and correctable by retargeting of backup weapons, and the case of no reprogramming where each missile is irrevocably committed to a single target from the beginning.

The determination of the measure of effectiveness, including these operation parameters, will now be considered. As before, let $\overline{S}$ denote the required damage criteria (the required minimum target survival probability). Let SSKP denote the single-shot kill probability for a successfully detonated weapon against the target class considered. The SSKP is, therefore, a function of the yield of the weapon, the delivery accuracy of the weapon, and the target vulnerability only and does not depend upon the other operational factors. Define the non-reprogrammable degradation factor, RN, as follows, depending upon the degree of reprogramming assumed:

$$RN = \begin{cases} Q \cdot RL \cdot RF \cdot RT \cdot P & \text{(no reprogramming)} \\ RF \cdot RT \cdot P & \text{(partial reprogramming)} \\ RT \cdot P & \text{("full" reprogramming)} \end{cases} \qquad (13)$$

RN, therefore, represents the fraction of the original force, for which no correctable failures have been detected, which will successfully detonate in the vicinity of the target. Let RR denote the reprogrammable degradation factor defined as follows:

$$RR = \begin{cases} 1 & \text{(no reprogramming)} \\ Q \cdot RL & \text{(partial reprogramming)} \\ Q \cdot RL \cdot RF & \text{("full" reprogramming)} \end{cases} \qquad (14)$$

The fraction of the original force which fails in such a way as to be detected, and for which standby weapons can be retargeted is, therefore, represented by $1 - RR$.

The effective single-shot survival probability of a target for a weapon whose failure is not detected or corrected is then given by

$$S = 1 - RN \cdot SSKP \qquad (15)$$

Having determined S, Equations (12) are then employed to determine[1] α, which is the average number of "successfully launched" missiles required to achieve the desired damage measures by $\overline{S}$. A "successfully launched" missile is any missile for which a failure has not been detected and corrected. The total number of missiles in inventory required to achieve this specified damage is then given by $\alpha' = \alpha/RR$, since RR represents the fraction of the total inventory which will be "successfully launched." Since the final measures of effectiveness desired are the reciprocal of α', the final result is:

$$MOE = RR/\alpha. \qquad (16)$$

In summary, the measure of effectiveness of any given weapon system against a particular target class is to be determined by the application, in the following order, of Equations (13), (14), (15), (12), and (16).

1. Note that α will depend upon which reprogramming case is pertinent.

APPENDIX "C" TO ENCLOSURE "A"

PARAMETRIC STUDY OF SURVIVABILITY
OF FIXED VS. MOBILE SYSTEMS

SECOND VOLUME OF

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PARAMETRIC STUDY OF SURVIVABILITY OF FIXED VS. MOBILE SYSTEMS

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FIGURES

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EFFECT OF INCREASING THE AREA OF OPERATION OF A MOBILE SYSTEM

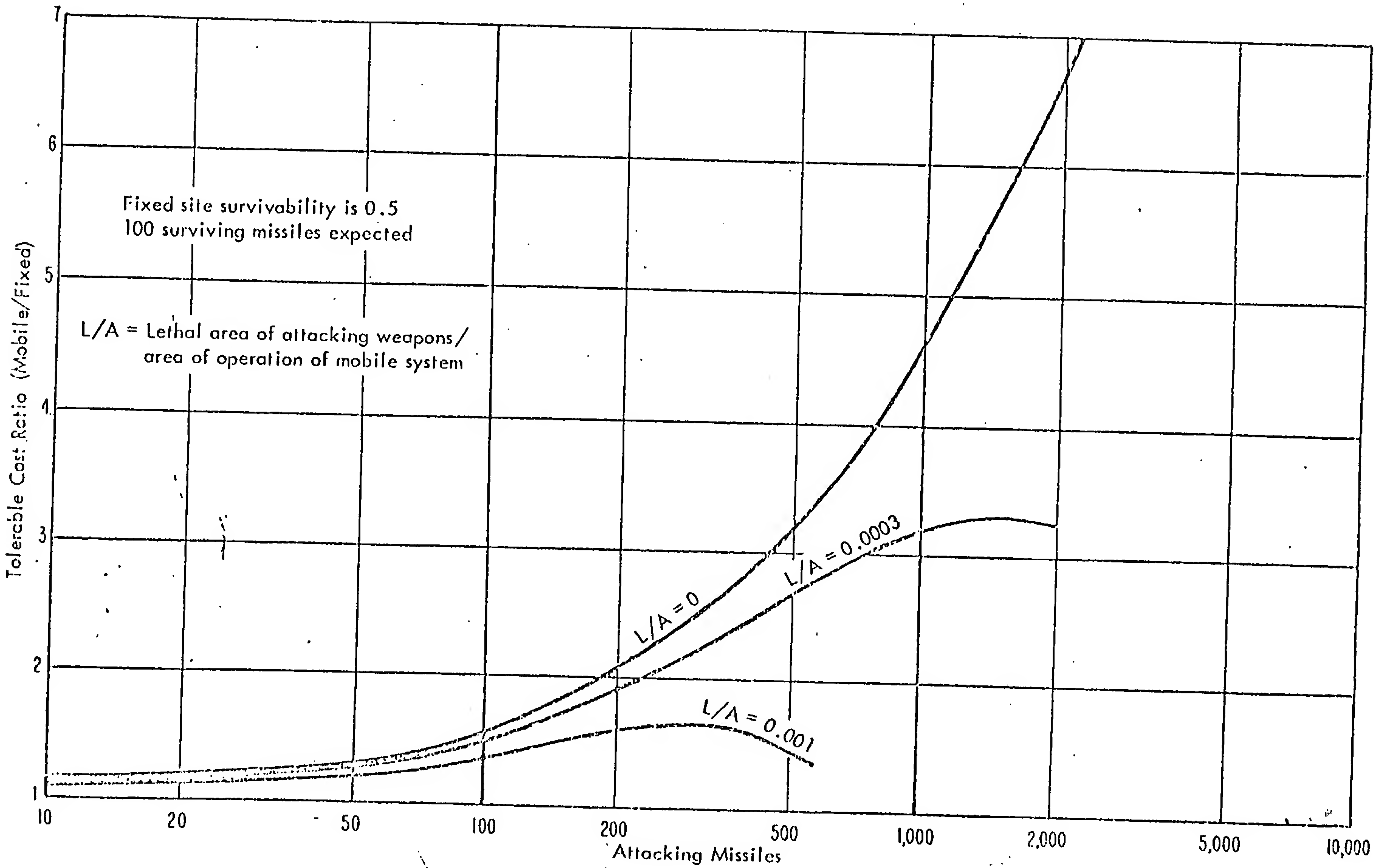

FIGURE 7
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AREA MOBILE CASE FOR INCREASED FIXED SITE SURVIVABILITY

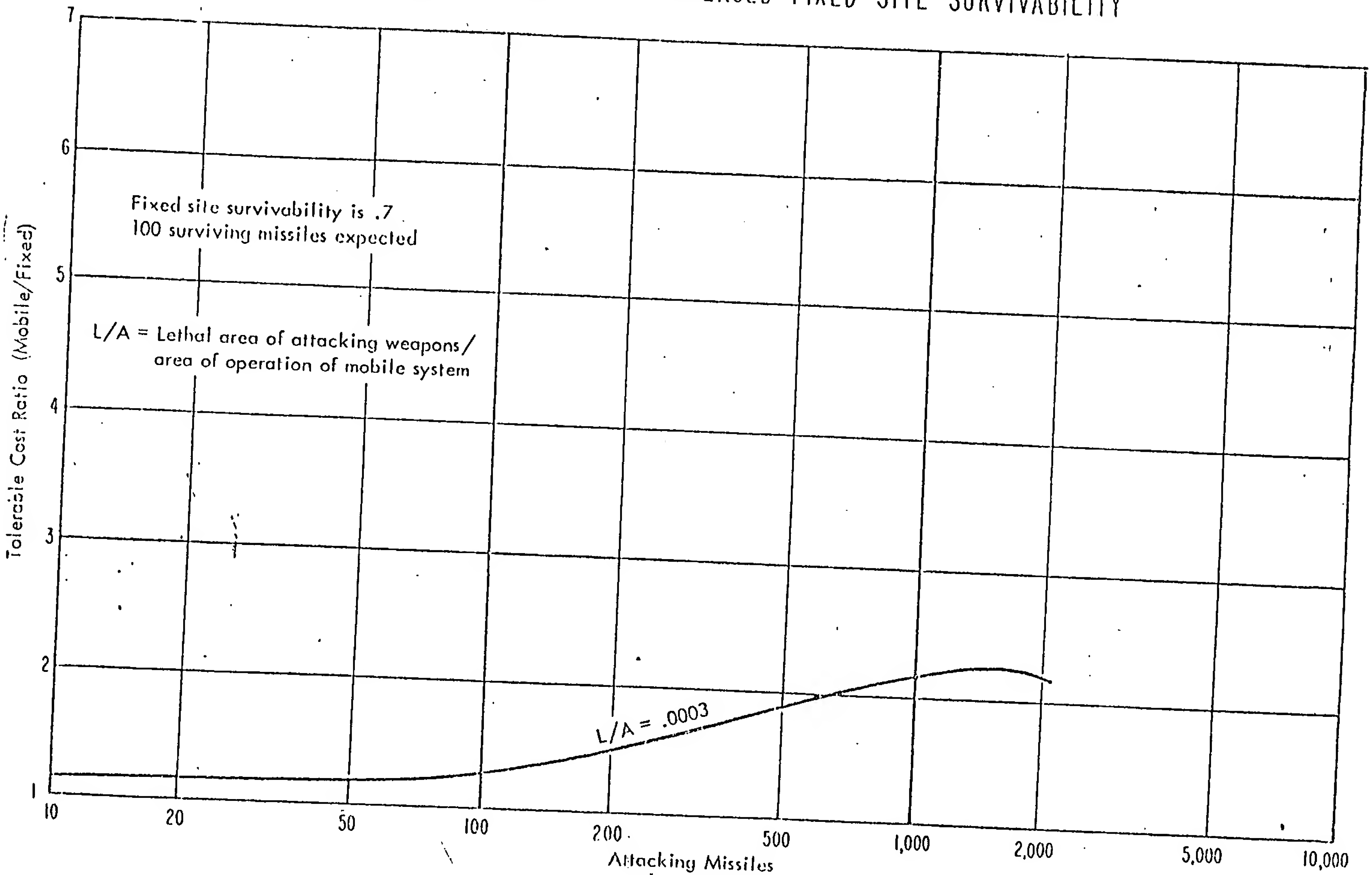

FIGURE 8
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as ratios of lethal lengths to total lengths of road or track for a linear mobile system. In the area mobile cases the dimensionless ratio of attacking missiles/expected surviving missiles cannot be used as before.

ANNEX TO APPENDIX "C"

DERIVATIONS OF INDEX COST RATIOS

ANNEX TO APPENDIX "C"

DERIVATION OF INDEX COST RATIOS

LIST OF SYMBOLS

NR = number of attacking missiles

NF = number of fixed missiles

NM = number of mobile missiles

SF = surviving fixed weapons expected

SM = surviving mobile weapons expected

β = NR/NF

δ = NR/SF

R_m = value of a single surviving mobile missile

R_f = value of a single surviving fixed missile

ρ = R_m/R_f

A = area within which a mobile system is known to operate

L = lethal area of enemy weapon

F = fraction of an area surviving an enemy attack (not covered to psi level desired

NS = number of possible firing sites for point-to-point mobile system

γ = NM/NS

β' = NR/NS

S_f = survivability of fixed site

S_m = survivability of mobile weapon at targeted launching site

C = index cost ratio.

DISCUSSION

1. If NR missiles are launched to cover as uniformly as possible a set of NF targets, then $NR - NF\left[\frac{NR}{NF}\right]$ 1/ of the targets receive an extra missile above the $\left[\frac{NR}{NF}\right]$ missiles assigned the other targets. The expected number of surviving targets, SF, is thus given by

$$SF = \left\{NR - NF\left[\frac{NR}{NF}\right]\right\} S_f^{\left[\frac{NR}{NF}\right]+1} + \left\{NF - NR + NF\left[\frac{NR}{NF}\right]\right\} S_f^{\left[\frac{NR}{NF}\right]} \quad (1.1)$$

1/ [ ] is used to indicate the largest integer contained in the bracketed expression.

where S_f is the single shot survivability of the target.[1]

To simplify notation, let $\beta = \frac{NR}{NF}$. If (1.1) is multiplied through by $\frac{NF}{NF}$, there is obtained

$$SF = NF\, S_f^{[\beta]} \left\{ (\beta - [\beta]) S_f + (1 - \beta + [\beta]) \right\} \quad (1.2)$$

which can be rearranged to give

$$SF = NF\, S_f^{[\beta]} \left\{ 1 - (1 - S_f)(\beta - [\beta]) \right\} \quad (1.3)$$

2. At this juncture a number of paths is possible. We have chosen to emphasize the surviving numbers of fixed or mobile missiles, deriving the cost ratio that must exist at parity--that is, as nearly equal as possible expected numbers of surviving missiles from either a fixed or a mobile force. This ratio, designated C, is given by NF/NM. C expresses the number of fixed missiles that must be purchasable at the price of one mobile missile. Alternatively, C expresses the maximum cost of mobile missiles relative to fixed missiles that can be tolerated (since mobile and fixed systems are being compared at parity in survivability). The higher the value of C, then, the more favorable appears the mobile system, since, by implication, the mobile system is sufficiently worthy in its survivability that the weapon buyer can stand higher and higher relative cost before switching his preference to the fixed system.

CASE 1 - INVULNERABLE MOBILE SYSTEM

3. If Equation (1.3) be divided by NR and the quotient $\frac{NR}{SF}$ be designated δ, there is obtained

$$\frac{1}{\delta} = \frac{1}{\beta} S_f^{[\beta]} \left\{ 1 - (1 - S_f)(\beta - [\beta]) \right\} \quad (3.1)$$

which can be rearranged to give δ/β as a function of β, that is

$$\frac{\delta}{\beta} = \frac{1 + \delta S_f^{[\beta]} (1 - S_f)}{S_f^{[\beta]} + [\beta] S_f^{[\beta]} (1 - S_f)} \quad (3.2)$$

[1] S_f may include a measure of the nonreprogrammable reliability of the enemy missile, R, i.e., $S_f = (1 - RP_K)$ where P_K is the kill probability.

Since $\frac{\delta}{\beta}$ equals $\left(\frac{NR}{SF}\right)\left(\frac{NF}{NR}\right)$ or $\frac{NF}{SF}$, since $SF = SM$ by the parity constraint, and since $SM = NM$ for an invulnerable mobile weapon system, $\frac{\delta}{\beta}$ is the desired ratio, $\frac{NF}{NM}$.

4. If it is considered that surviving fixed and mobile weapons may not be of identical characteristics, differing, perhaps, in reliability, a further parameter, ρ, may be introduced at this point.

5. ρ expresses the relative values of surviving mobile to surviving fixed weapons. The final expression for the cost ratio, which we will denote C1, is then

$$C1 = \rho \frac{1 + \delta S_f^{[\beta]}(1 - S_f)}{S_f^{[\beta]} + [\beta] S_f^{[\beta]}(1 - S_f)} \tag{5.1}$$

6. This equation is susceptible to single computer solution for various values of δ and S_f, given the proper integral value for $[\beta]$. $[\beta]$ itself is rather easily determined. Recall Equation (3.1)

$$\frac{1}{\delta} = \frac{1}{\beta} S_f^{[\beta]} \left\{ 1 - (1 - S_f)(\beta - [\beta]) \right\}$$

7. The right-hand side of this equation is a monotone decreasing function of β and reduces to $\frac{1}{J^{\beta}} S_f^{[\beta]}$ at integral values of β. If then, the function $\frac{1}{J} S_f^{J} - \frac{1}{\delta}$ changes sign, becoming negative, between integral values J and $J + 1$, the exact value of $[\beta]$ is given by J.

8. Since $[\beta]$, as shown above, is a function of (S_f, δ), then $C1 = F(S_f, \delta)$ only.[1]

[1] ρ is merely a multiplicative factor throughout and is carried for completeness.

CASE 2 - AREA-MOBILE SYSTEM

9. If the precise locations of elements of a mobile system are unknown except within a relatively large area, the optimal enemy tactic is to attempt uniform coverage of the large area to the extent that his weapon stockpile permits (granting that he desires to attack the mobile system). The expected fraction of the mobile system destroyed is then the fraction of the area covered. Since a mobile system is generally somewhat soft, the lethal radius of an enemy weapon will usually far exceed its CEP. For this reason, it appears likely that an enemy could essentially pattern his weapons in a large area, achieving additive coverage for each weapon until his total coverage exceeded some 50 or 60 percent (one layer of rather conservatively spaced weapons to eliminate any appreciable overlap). We can express the fraction, P, of an area, A, surviving under this rule as

$$P = 1 - \frac{L \cdot NR}{A} \quad (1 - P < 0.6) \qquad (9.1)$$

where L is the lethal area of a single weapon.[1] As a worst possible case, the enemy might be considered to resort to random attack in the area, leaving a fraction surviving that can be approximated by

$$P = \left(1 - \frac{L}{A}\right)^{NR} \qquad (9.2)$$

10. In the main section of this paper we have presented results only for the former case (Equation (9.1)) which seems much more realistic for the general problem.

11. Recalling that $C1 = \rho\ddot{u}/\beta = \rho \cdot NF/SF = \rho \cdot NF/SM$, and that $SM = NM \cdot P$, we obtain

$$C2 \equiv \rho\frac{NF}{NM} = C1 \cdot P \qquad (11.1)$$

where P is (preferably) given by Equation (9.1).

[1] In cases investigated, the constraint of about 50 to 60 percent total coverage did not occur at sufficiently low levels to prohibit interesting results.

12. C2 is now dependent on the explicit enemy force level, NR, in addition to the dimensionless parameter $\delta = \frac{NR}{SF}$. Hence, $C2 = F(\delta, S_f, NR, L/A) = F(SF, NR, S_f, L/A)$.

CASE 2A - LINEAR MOBILE SYSTEM

13. Certain types of mobile systems may operate in a manner that precludes the conceptual use of the area mobile model just described (Case 2). Rail or some road mobile systems may be treated as linear, rather than area mobile systems. The attacker attempts to blanket the track or road network employed by the mobile system achieving a portion of the system destroyed given by $1 - \frac{LL \cdot NR}{TL}$, where LL is a "lethal length" and TL the total length of road or rail upon which the system is known to operate. The "lethal length" LL, is a function of yield and CEP, but for reasonably low CEP's may be approximated by twice the lethal radius.

14. It is apparent that we can write a new formula for a linear mobile system patterned after Equation (11.1) for the area mobile system:

$$C2A = C1 \cdot P' \tag{14.1}$$

where $$P' = 1 - \frac{LL \cdot NR}{TL}.$$

15. The identication of L with LL and A with TL is obvious. There is one additional feature of an attack on a linear mobile system. The constraint of less than about 60 percent coverage for validity of Equation (9.1) no longer applies. Blanketing of a line target does not involve the necessary overlapping of circular weapon effect patterns required to blanket an area target beyond about 60 percent. Hence, the upper limit of validity for formula (14.1) is well beyond $1 - P' = 60\%$.

16. If a mobile system utilizes presurveyed firing sites, moving more or less randomly among a number of such sites, the enemy, lacking precise intelligence on the instantaneous location of weapons, can only associate with each site a probability that a mobile weapon will be there. This probability is given simply as $\gamma = \frac{NM}{NS}$, where NS is the total number of firing sites the mobile system may employ. If the system can "fire from anywhere" but is restricted to a known area, Case 2 applies. If the area is unknown or too large for attack, Case 1 applies.

17. If a mobile weapon (with its associated mode of transport) be located at a targeted site, the mobile weapon might still survive an enemy weapon with probability S_m. Since, in general, portions of a mobile system will be in transit at any time, that fraction of the force is guaranteed to survive (barring a concommitant area attack). Such a system is really a mixed system of invulnerable and vulnerable mobile elements and therefore the higher the fraction in transit the better appears the mobile system against any attack.[1]

18. For Case 3, we shall consider a particularly soft mobile weapons system and correspondingly high enemy missile post-launch reliability and kill probability, combining to give a value of S_m essentially negligible.

19. If we recall again that $\beta/\delta = \left(\frac{NR}{NF}\right)\left(\frac{SF}{NR}\right) = \frac{SF}{NF} = \frac{SM}{NF}$, we can write

$$NF = \frac{\delta}{\beta} SM. \tag{19.1}$$

1/ In no case can the index C exceed that for the invulnerable mobile system. If the mobile system is constrained to fire only from presurveyed sites, then destruction of all sites will insure the neutralization (at least for some time) of even the portion of the system in transit at the time of attack.

, SM can be expressed in a manner analogous to Equation (1.3) for SF, that is

$$SM = NM\, S_m^{[\beta']} \left\{ 1 - (1 - S_m)(\beta' - [\beta']) \right\} \qquad (19.2)$$

but β' is now NR/NS. In terms of $\frac{C3}{\rho}$ $\frac{NF}{NM}$, $\beta = \frac{NR}{NF}$, and $\gamma = \frac{NM}{NS}$, β' becomes $\frac{C3\,\beta\gamma}{\rho}$.

20. Case 4 (treated in the following section) utilizes the complete Equation (19.2), $(S_m \neq 0)$, Case 3 considers the reduced equation when S_m is set equal to zero and the enemy utilizes at most, therefore, only a single layer of weapons $([\beta'] = 0)$.

$$SM = NM\left(1 - \frac{C3}{\rho} \cdot \beta\gamma\right) \qquad (20.1)$$

21. Substituting in Equation (19.1) and multiplying by ρ, we obtain

$$\rho\, NF = \rho \frac{\delta}{\beta} NM\left(1 - \beta\gamma \cdot \frac{C3}{\rho}\right) \qquad (21.1)$$

which can be rearranged, noting that $\rho \frac{NF}{NM} = C3$ and $\rho \cdot \frac{\delta}{\beta} = C1$, to yield

$$C3 = C^1\left(1 - \frac{C3}{\rho} \cdot \beta\gamma\right) \qquad (21.2)$$

or

$$C3 = \frac{C1}{1 + \frac{C1 \cdot \beta\gamma}{\rho}} \qquad (21.3)$$

which can be simplified further, since $C1 = \rho \frac{\delta}{\beta}$, to give the final expression

$$C3 = \frac{C1}{1 + \delta\gamma} \qquad (21.4)$$

Therefore, $C3 = F(S_f, \delta, \gamma)$ only.

CASE 4 - POINT-TO-POINT MOBILE SYSTEM $(S_m \neq 0)$

22. As in Case 3, we proceed from Equation (19.2), retaining all terms,

$$SM = NM\, S_m^{[\beta']} \left\{ 1 - (1 - S_m)(\beta' - [\beta']) \right\}$$

23. As in Case 1, we may solve for the cost ratio in terms of other parameters obtaining finally

$$C4 = C1 \cdot \frac{S_m^{[\beta']} + [\beta'] S_m^{[\beta']} (1 - S_m)}{1 + \gamma\delta S_m^{[\beta']} (1 - S_m)} \qquad (23.1)$$

This equation was solved for C4 by calculating trial values of C4, say $\overline{C4}$, by substitution of successive integral values of $[\beta']$, say j, until $\left[\overline{C4}\,\beta\gamma\right] = j$. Then $C4 = \overline{C4}$. It might be noted that $C4 = F(S_m, S_f, \delta, \gamma)$ since $\beta = F(C1, \delta) = F(S_f, \delta)$ as shown previously.

SUMMARY

24. In Case 2 only (Area-Mobile System) does the functional relation for the cost ratio demand a specification of the attacking force level, NR (and in addition, L/A). In all other situations, the dimensionless ratio, $\delta = \frac{NR}{SF} = \frac{NR}{SM}$ specifies the case. Secondary variations are obtained by choice of γ, S_f, and S_m. If ρ is felt to differ from 1, the indicated cost ratio C should be adjusted to ρC.

25. For convenience, the basic equations are summarized below.

$$C1 = \rho \cdot \frac{1 + \delta S_f^{[\beta]} (1 - S_f)}{S_f^{[\beta]} + [\beta] S_f^{[\beta]} (1 - S_f)} \qquad \text{Invulnerable Mobile} \qquad (25.1)$$

$$C2 = C1 \cdot P \qquad \text{Area-Mobile} \qquad (25.2)$$

$$C3 = \frac{C1}{1 + \delta\gamma} \qquad \text{Point-Mobile } S_m = 0 \qquad (25.3)$$

$$C4 = C1 \cdot \frac{S_m^{[\beta']} + [\beta'] S_m^{[\beta']} (1 - S_m)}{1 + \gamma\delta S_m^{[\beta']} (1 - S_m)} \qquad \text{Point-Mobile } S_m > 0. \qquad (25.4)$$

CASE 5 - INVULNERABLE MOBILE VS. LINEAR MOBILE SYSTEMS

25. Since all previous cost ratios derived are similar in relating various mobile systems to the fixed system, there exist further cost ratios not yet explicitly derived which relate the various types of mobile systems to each other. The present case derives the tolerable cost ratio that must obtain for equal numbers of surviving missiles from an invulnerable mobile system compared to a linear mobile system.

26. If Equation (14.1) for the linear mobile system be recalled

$$C2A = C1 \cdot P', \tag{26.1}$$

and explicit relations for C2A and C1 be substituted, there is obtained

$$\frac{NF}{NM} = \rho \frac{NF}{SM} \cdot P' \tag{26.2}$$

The comparison base can now be changed from the fixed system to the invulnerable mobile system simply by changing NF to NM′, where NM′ refers to the invulnerable mobile system

$$\frac{NM'}{NM} = \rho \frac{NM'}{SM} \cdot P' \tag{26.3}$$

27. As usual NM′ = SM′ for the invulnerable system, where SM′ is the surviving mobile weapons from the invulnerable system.

$$\frac{NM'}{NM} = \rho \frac{SM'}{SM} \cdot P' \tag{27.1}$$

If C5 be taken as NM/NM′, we thus obtain for the usual condition SM′ = SM.

$$C5 = \frac{1}{\rho \cdot P'} \tag{27.2}$$

APPENDIX "C" TO ENCLOSURE "A"

PARAMETRIC STUDY OF SURVIVABILITY OF FIXED VS. MOBILE SYSTEMS

PURPOSE

1. To investigate the conditions under which fixed or mobile systems are preferred on the basis of their relative costs and their survivability to enemy missile attack.[1/] Results of this Appendix will be integrated with other pertinent variables in the Main Paper of Enclosure "A".

METHOD

2. Analytic models are derived permitting parametric comparison of fixed versus mobile systems. The results are then applied to the MINUTEMAN and POLARIS systems.

ASSUMPTIONS

3. It is assumed that no missiles are launched during the enemy ballistic missile attack. They are compared on the basis of their survivability to the entire enemy attack.

4. The attacker is given credit for no intelligence on the precise location of mobile weapons at the time of his attack.

DISCUSSION

INTRODUCTION

5. The models developed here assume that one prefers either an all-fixed or an all-mobile system, and investigate the conditions under which each is preferred. It is usually true that if conditions are specified precisely then a single system rather than a mix is optimum, in the mathematical sense. There are some

1/ Consideration of cost and survivability alone are sufficient for comparison of two systems provided all other pertinent variables (such as yield, CEP and reliability) are equal. If they are not equal then the analysis is appropriate for evaluation only if cost is replaced by an appropriate cost/effectiveness index.

narrow ranges of conditions and special types of mobile systems for which the mathematical optimum is a mixture. Furthermore, if one is uncertain as to the characteristics of the enemy attack, he may assign weights to various enemy capabilities or attack levels and so arrive at a mixed system. However, as will become apparent shortly, there exist wide ranges of enemy capability and attack levels for which either the mobile or fixed system is always optimal.

6. Another line of reasoning can also lead to a mixed system. If different uses are to be made of surviving weapons--different target types, etc.--then different types of weapons may be needed, each designed for its specific task. Such a mixture is generally a cheaper alternative than the design of a universal all-purpose weapon. Similarly, if a superior system is available later than others, a mix may result.

7. In the following discussion, we shall restrict ourselves to essentially similar systems designed for the same objective, and consider the threats or ranges of threats under which fixed or mobile alternatives (but not both) are the best buy. This approach is adequate for discussing the relative merits of fixed and mobile systems, but does not answer the question of an optimum force program.

TYPES OF FIXED AND MOBILE SYSTEMS

8. From the attacker's point of view, the kill probability of his weapons might be thought to consist of:

1. The chance that his weapon destroy a given target at its aiming point, and

2. The chance that the desired target is sufficiently near the aiming point.

9. The fixed system usually uses hardening to reduce the first alternative probability. The mobile system, on the other hand, attains its "security" by lowering the second probability. That is, the multiplicity of points from which the mobile system can fire actually diffuses the enemy attack over a larger area, resulting in lower densities of attack and correspondingly higher survivability at any given point.

10. In the following discussion, then, we will consider fixed systems of relatively high survivability, due to hardening, and mobile systems of relatively low survivability when located near an aiming point for an enemy weapon. Such mobile systems will be divided into two types. The first type (called "point-to-point mobile") achieves its survivability by the multiplicity of firing sites that it can employ. The expected number of mobile missiles at a given firing site is simply the total number of mobile missiles divided by the total number of possible firing sites for the mobile system. As the number of firing sites per missile increases, of course, the survivability of the mobile systems increases. When more missiles are added to this system more firing sites are also prepared.

11. The second type of mobile system is constrained by the area in which it must operate. Within this area, the number of permissible firing points is assumed to be greater than the number of enemy aiming points for coverage of the entire area. As the area of the operation of the mobile system increases, its survivability correspondingly will gain. When more missiles are added to this system they are inserted into the constrained area. A subclass of the area-mobile system is the linear-mobile system. A rail or road-mobile system might be thought best approximated

by a line or series of lines along which the system possesses a fire-from-anywhere capability.[1]

12. If the number of independent[2] firing points could increase without bound in the first case, the mobile system would become essentially invulnerable since the enemy attack would be infinitely diluted. In the second case, if the area of operation approaches infinity the enemy attack is correspondingly diluted and the vulnerability of the mobile system reduced to zero.

METHOD OF COMPARISON

13. In each case mentioned above we have derived an index that permits comparison of mobile versus fixed systems. This index, which is a function of the number of enemy missiles to be used in a counterforce role against the fixed or mobile system, expresses the price ratio (mobile to fixed) at which it is indifferent which system is preferred. If the mobile system costs more than the index implies, then the fixed system is preferred. The mobile system is preferred if its cost is thought to be any lower value. Thus, in general the cost ratio index that is derived is some measure of the "goodness" of the mobile system relative to the fixed system that it is compared against. The higher this index the more the mobile system is to be preferred. If the index drops below the expected actual cost ratio for a pair of systems, then the fixed system is preferred. In the figures that follow, various factors that enter into the comparison of fixed versus mobile systems are explicitly illustrated. These factors are considered as principal ingredients that influence the survivability of either system. It should be noted that factors such as yield and CEP of enemy weapons are

1/ Results given for the area-mobile system may be applied directly to any linear-mobile system by considering the "area" as a total "length" of rail track or road. The lethal area of the enemy weapon then becomes a lethal length (about twice the kill radius).

2/ Two firing points are independent if they are spaced sufficiently far apart that both cannot be destroyed by a single enemy missile.

important only insofar as they combine to influence the survivability of individual points that might be targeted when hard weapons or soft mobile weapons are located at these points.

PRESENTATION OF RESULTS

14. The Annex to this Appendix derives equations for determining the index cost ratios as a function of type mobile system, hardness of fixed system, and enemy threat. A variety of special cases are presented in the figures below.

Point-to-Point Mobile Systems

15. Figures 1 to 3 show the effect of changes in the ratio γ of total mobile missiles to total number of possible independent firing points in a point-to-point mobile missile system. In all figures "survival probability" values are single-shot survivabilities. As the number of firing points increases, the ratio γ decreases and we are led to situations in which mobile systems appear more and more favorable. This is, of course, reasonable. The upper curve, where γ equals 0, applies to the case of an invulnerable mobile system. In this case, beyond certain low attacking levels, the index cost ratio quickly rises. It should be noted that the invulnerable mobile system is preferred to the fixed system for all Soviet force levels beyond the intersection of the curve with the horizontal line representing the estimated cost ratio. This intersection, of course, occurs at different attacking force levels depending on just what one estimates for the relative cost of mobile/fixed weapons. The various point-to-point mobile systems, characterized by values of γ greater than 0, exhibit maxima as may be seen in the figures. Thus, the estimated cost ratio may intersect the curve twice. At force levels of attacking weapons beyond the second intersection the fixed system becomes more attractive. If the estimated ratio is above the maximum the fixed system is preferred regardless of the enemy force level.

FIGURES 1, 2 AND 3

EFFECT OF VARIATIONS IN γ

EFFECT OF VARIATIONS IN γ OF POINT MOBILE SYSTEMS

MOBILE FORCES OF "POINT-TO-POINT" TYPE

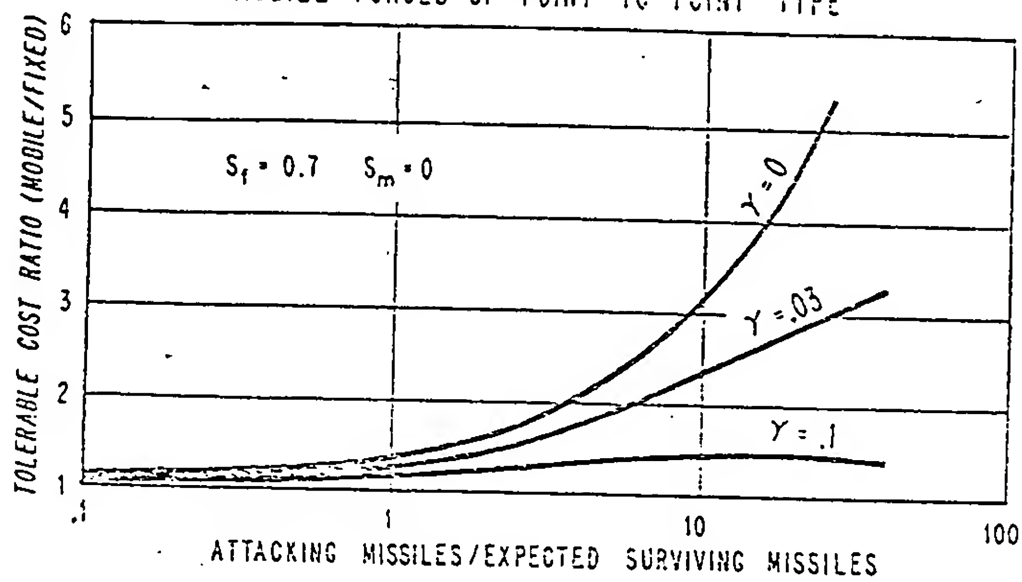

FIGURE 1

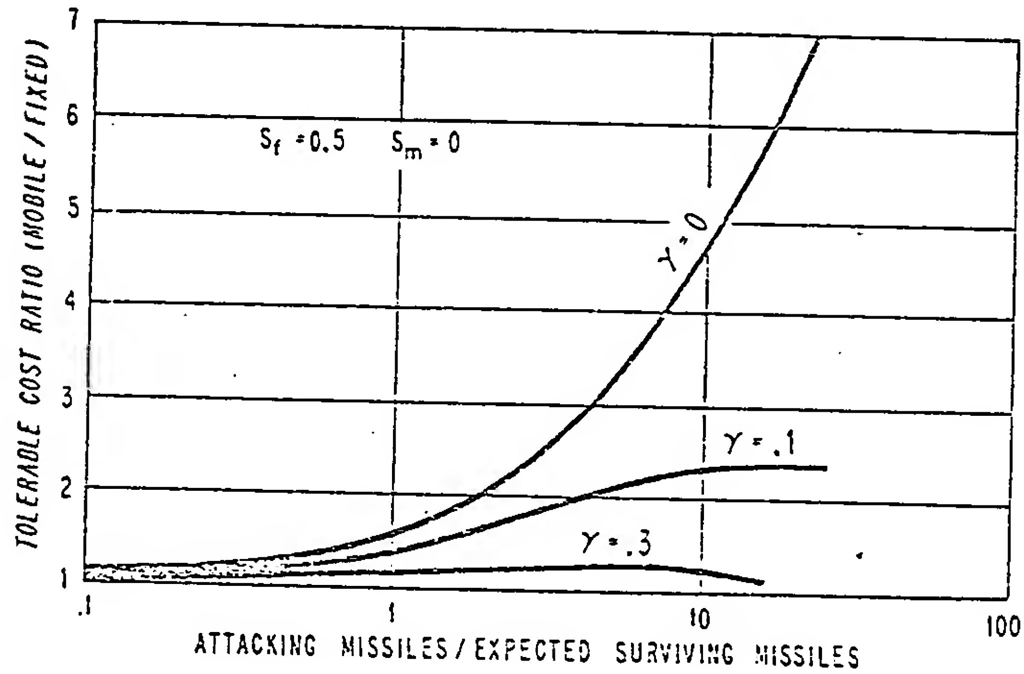

FIGURE 2

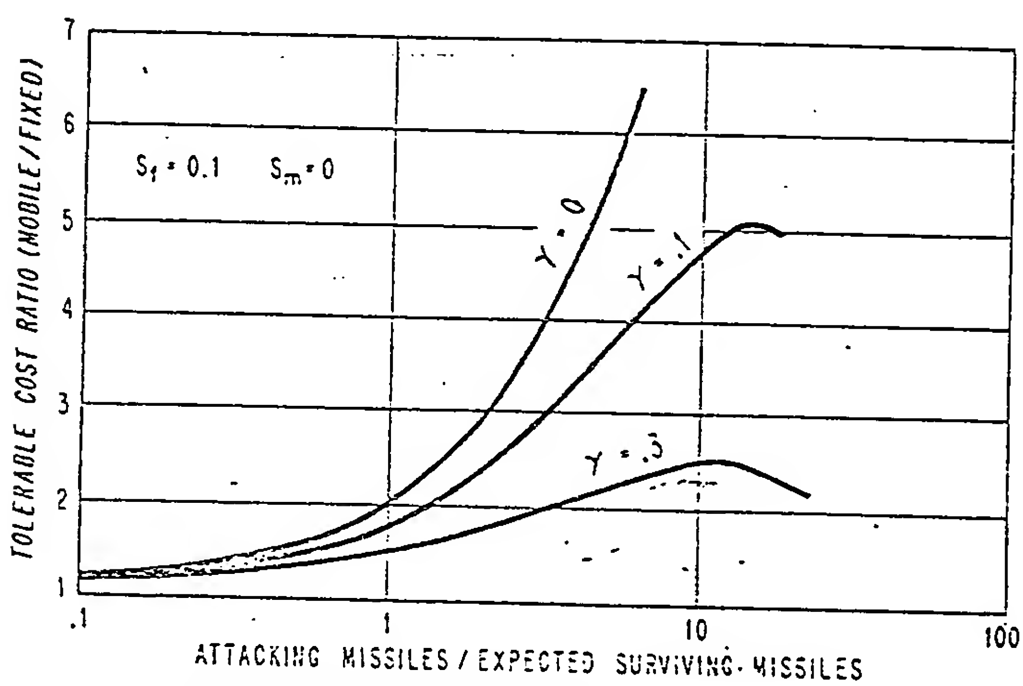

FIGURE 3

S_m = SURVIVABILITY OF MOBILE WEAPON AT A FIRING SITE

S_f = SURVIVABILITY OF FIXED SITE

$\gamma = \frac{\text{TOTAL MOBILE MISSILES}}{\text{TOTAL POSSIBLE FIRING SITES}}$

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FIGURES 1, 2 & 3
APPENDIX "C" TO
ENCLOSURE "A"
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16. The effect of variations in fixed-site survivability may also be seen in these first three figures. Index cost ratios always become more and more favorable for the fixed system as the survivability of the fixed system increases.

17. These figures also demonstrate that, for the invulnerable mobile system, the fewer the required number of survivors the higher the number of attacking missiles per survivor and hence the higher the cost ratio that can be tolerated for mobile weapons relative to fixed weapons. It is to be noted, however, that if the mobile system is truly invulnerable the enemy will allocate no weapons to it, but will transfer these weapons to other targets.

18. In all of the preceding discussion it is assumed that a mobile weapon, if located at a point where the attacker aims, is always destroyed. If the survivability of a mobile weapon be greater than zero, then, of course, the cost ratio that can be tolerated for the mobile system relative to the fixed system is higher. Figures 4 and 5 illustrate the manner in which this index cost ratio is dependent on the survivability of a mobile weapon when the weapon is located at one of the enemy aim points. It may be observed in these figures that the survivability can be quite influential in determining the cost ratio. The greater the survivability of the mobile weapon the higher the tolerable cost ratio becomes.

Area Mobile System

19. A second type of mobile system, the area (or linear) mobile system, presents certain aspects that differ from the point-to-point mobile system. The ratio of lethal area of the enemy weapon to the total area of operation of the mobile system is a parameter of primary interest. The optimal enemy tactic, in the absence of precise intelligence, is to attempt to blanket the area of operation of the mobile system. Since it is only the ratio of lethal area to total area that determines the cost ratio, only this ratio is shown in Figures 6, 7 and 8. The ratios may also be interpreted

FIGURES 4 AND 5

EFFECT OF CHANGES IN S_m

FIGURES 6 AND 7

EFFECT OF INCREASING THE AREA OF OPERATION OF A MOBILE SYSTEM

FIGURE 8

AREA MOBILE CASE FOR INCREASED FIXED SITE SURVIVABILITY

EFFECT OF CHANGES IN Sm

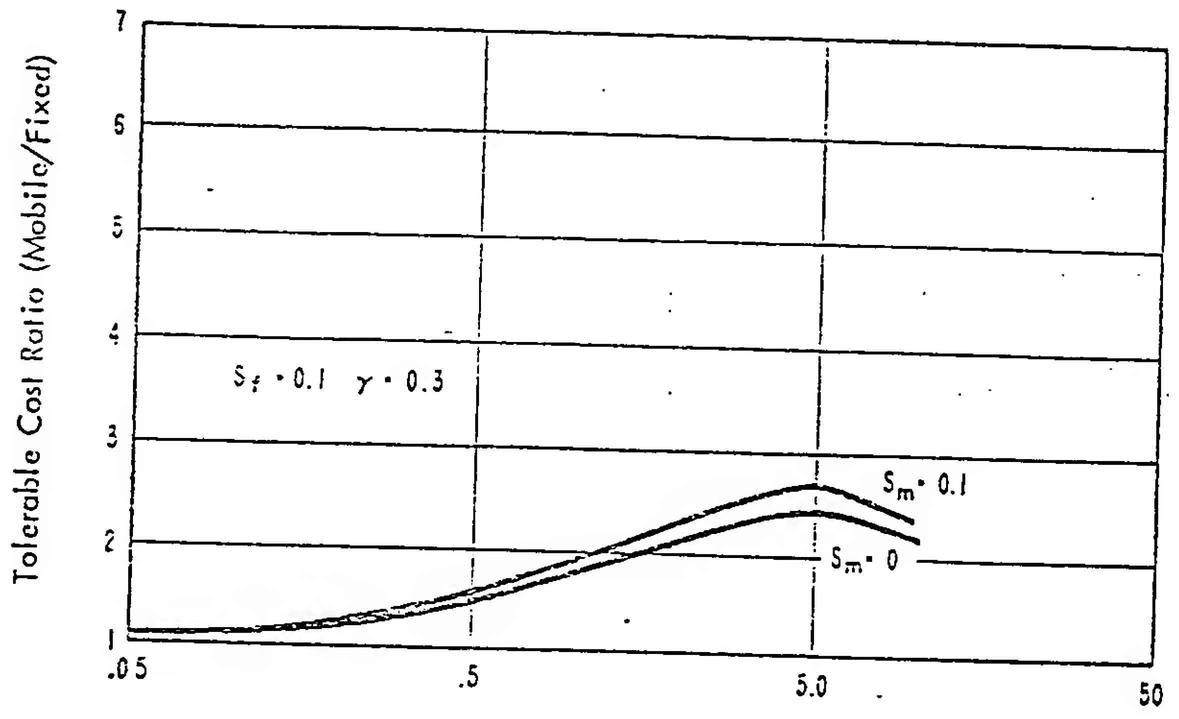

FIGURE 4

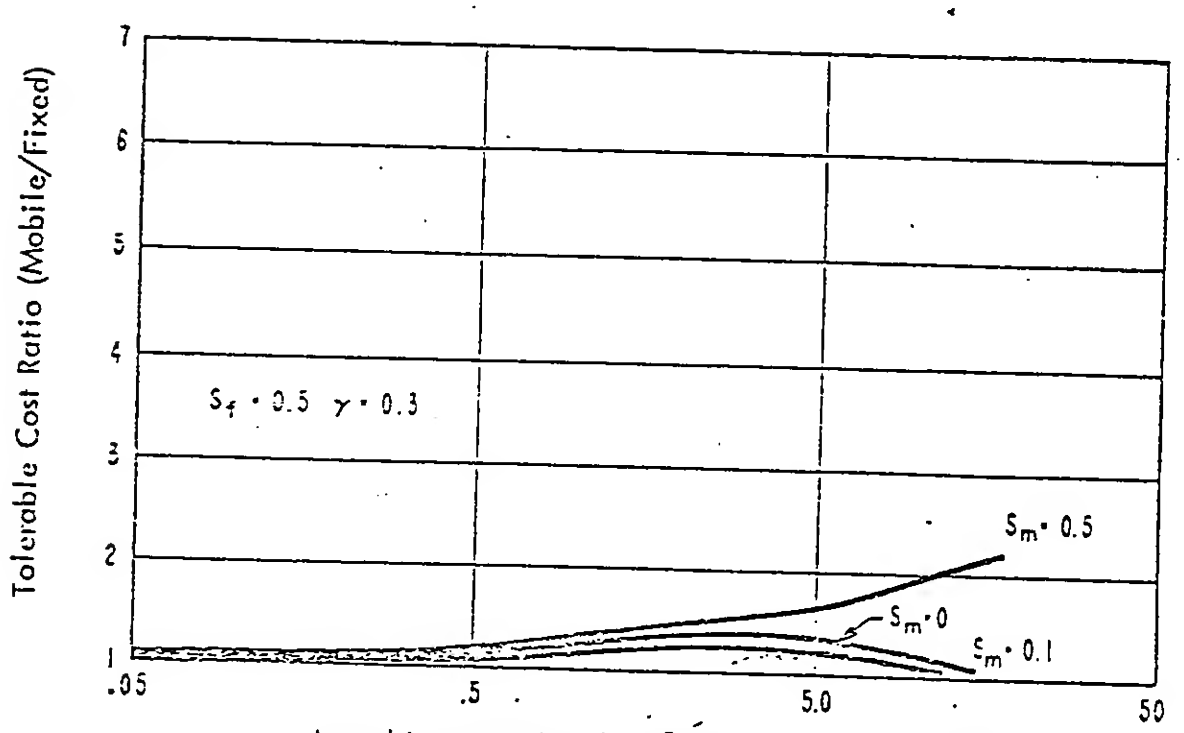

FIGURE 5

S_m = SURVIVABILITY OF MOBILE WEAPON AT A FIRING SITE

S_f = SURVIVABILITY OF FIXED SITE

$\gamma = \dfrac{\text{TOTAL MOBILE MISSILES}}{\text{TOTAL POSSIBLE FIRING SITES}}$

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FIGURES 4 & 5
APPENDIX "C" TO
ENCLOSURE "A"
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EFFECT OF INCREASING THE AREA OF OPERATION OF A MOBILE SYSTEM

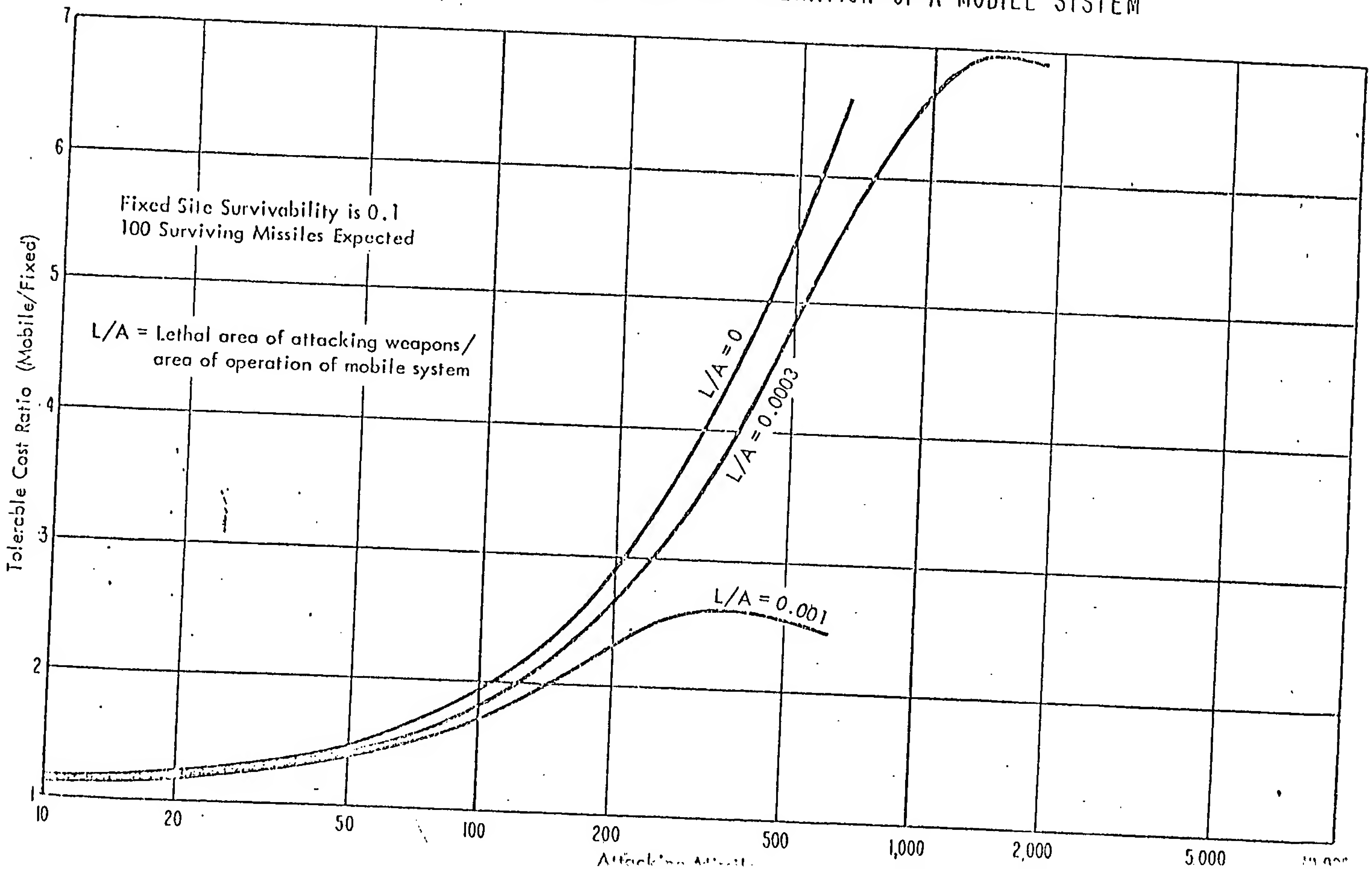

FIGURE 6
APPENDIX "C" TO
ENCLOSURE "A"
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